MAE 656 - Advanced Computer Aided Design

05. Shells and Membranes – Doc 01

Introduction

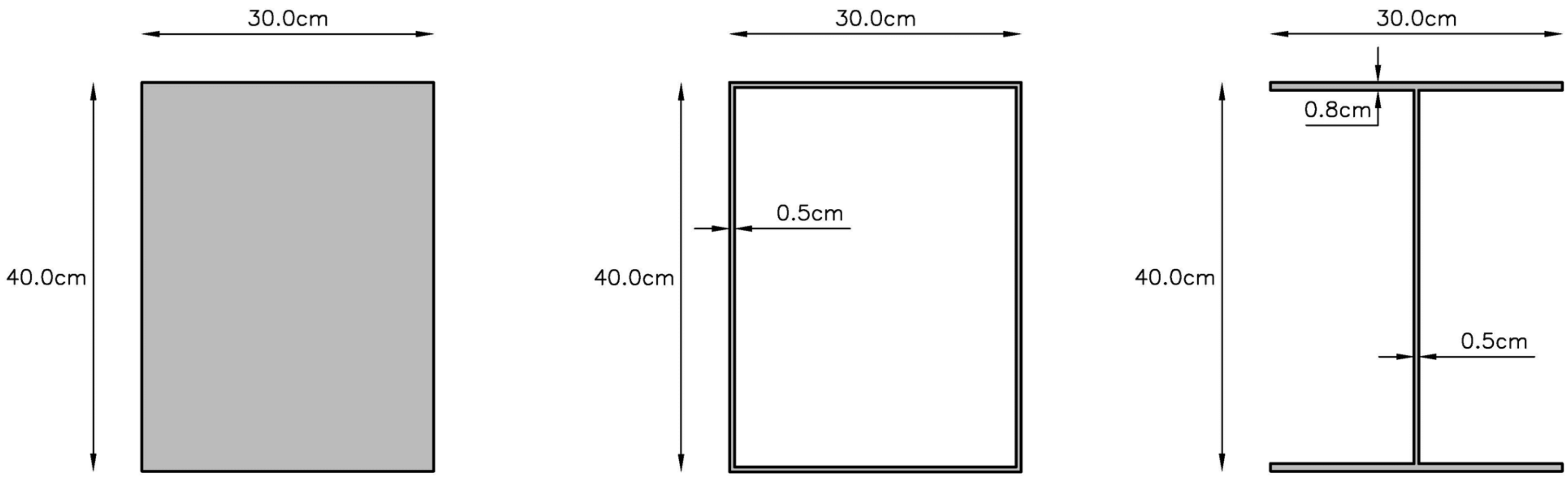
Introduction

Most of the structures used in engineering are made of thin parts or components:

Introduction

If a thin structure is well designed, it will use less material than a solid structure.

Less material implies less weight and less cost!



A	= 1200cm^2	A	= 69cm^2	A	= 67.2cm^2
I	= 160,000cm^4	I	= 16,645.7cm^4	I	= 20,801.5cm^4
W	= 942.0 kg/m	W	= 54.17 kg/m	W	= 52.75 kg/m
I/W	**= 169.85**	**I/W**	**= 307.32**	**I/W**	**= 394.33**

Simulation of thin wall structures

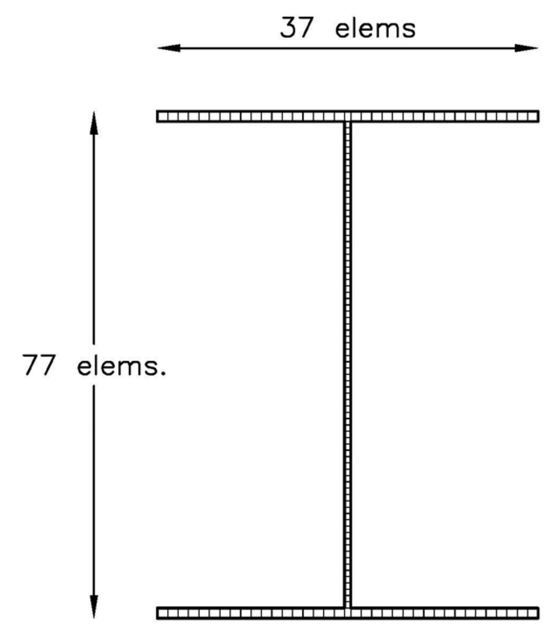
Having seen the advantage of using thin structures, if we use the simulation tools that we have seen so far to simulate them the computational cost may be impossible:

In previous beam, if we want to use regular hexahedral elements, we need a total of:

(37 + 37 + 77) elements
for each 0.8 cm of beam

To simulate a 1.0m of beam:

151 x 125 = 18,875 elements !!!

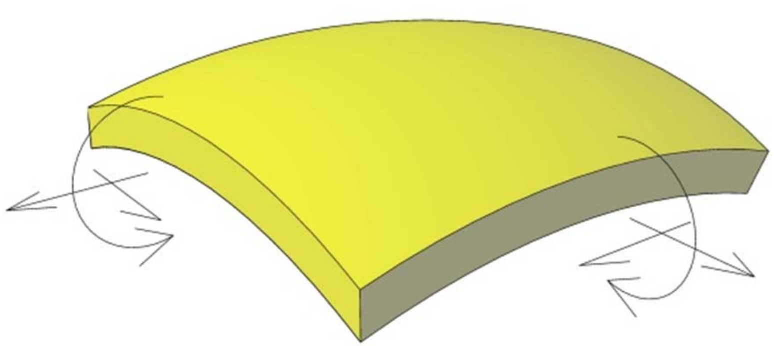


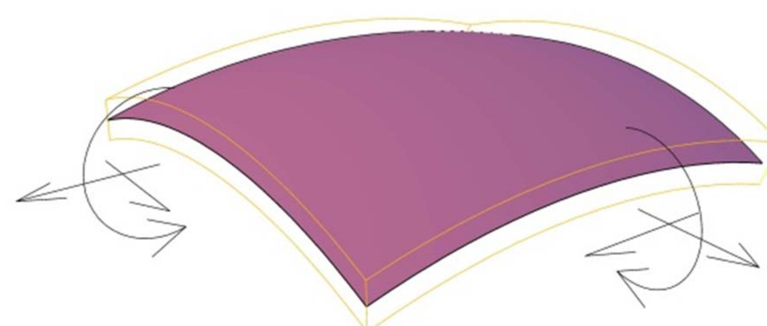
Simulation of thin wall structures

The best option to make this calculation affordable is to eliminate the thickness dimension from the calculation (as this is the one that forces to use such small elements).

Can we do it?

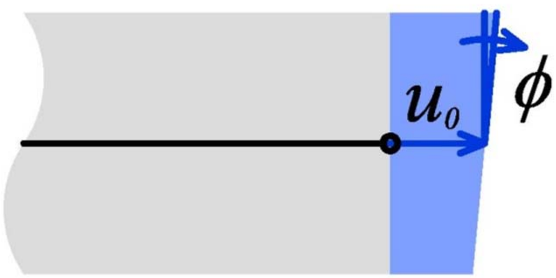
Yes. If we condensate the lamina deformation to its mid-plane:





Kinematics

Assuming that a plane cross section remains planar after deformation, the shell kinematics can be written as a function of the deformation of its mid-plane and the rotation of the shell:



$$
\begin{cases}
u(x,y,z) = u_0(x,y) - z \cdot \phi_x(x,y) \\
v(x,y,z) = v_0(x,y) - z \cdot \phi_y(x,y) \\
w(x,y,z) = w_0(x,y)
\end{cases}
$$

Strains

This displacement field provides the following strains:

$$
\begin{cases}
\varepsilon_x = \dfrac{\partial u}{\partial x} = \dfrac{\partial u_0}{\partial x} - z \cdot \dfrac{\partial \phi_x}{\partial x} \\[2ex]
\varepsilon_y = \dfrac{\partial v}{\partial y} = \dfrac{\partial v_0}{\partial y} - z \cdot \dfrac{\partial \phi_y}{\partial y} \\[2ex]
\varepsilon_z = \dfrac{\partial w}{\partial z} = \dfrac{\partial w_0}{\partial z} \\[2ex]
\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} - z \cdot \left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \right) \\[2ex]
\gamma_{xz} = \dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z} = \dfrac{\partial w_0}{\partial x} - \phi_x \\[2ex]
\gamma_{yz} = \dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z} = \dfrac{\partial w_0}{\partial y} - \phi_y
\end{cases}
$$

Strains

The strain field can be written in compact form as:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \end{bmatrix} + z \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Deformations γ_{xz} and γ_{yz} are considered zero when the thickness is sufficiently small, as distortion effects can be neglected.

Curvatures are calculated as:

$$\begin{cases} \kappa_x = -\dfrac{\partial^2 w_0}{\partial x^2} \\ \kappa_y = -\dfrac{\partial^2 w_0}{\partial y^2} \\ \kappa_{xy} = -2 \cdot \dfrac{\partial^2 w_0}{\partial x \cdot \partial y} \end{cases}$$

Stresses

Therefore, knowing the deformations and curvatures of the shell mid-plane, it is possible to calculate the deformation of any point in the laminate.

Using the stiffness matrix of the material, it is possible to obtain the stress field in each point:

$$\sigma = C \cdot \varepsilon$$

$$C = \frac{1}{1 - \nu_{xy} \cdot \nu_{yx}} \cdot \begin{bmatrix} E_x & \nu_{xy} \cdot E_x & 0 \\ \nu_{yx} \cdot E_y & E_y & 0 \\ 0 & 0 & (1 - \nu_{xy} \cdot \nu_{yx}) \cdot G_{xy} \end{bmatrix}$$

Generalized Stresses

The integration of the laminate stresses along the thickness results in the generalized stresses:

Membrane Stresses
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz$$

Bending Stresses
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \cdot z\, dz$$

Generalized Stresses

Replacing the stress by the strains in each point we obtain:

Membrane Stresses

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} dz$$

$$= \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot \left(\begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \end{bmatrix} + z \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) dz$$

Bending Stresses

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \cdot z \, dz$$

$$= \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot \left(\begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \end{bmatrix} + z \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) \cdot z \, dz$$

Generalized Stresses

The relation between the generalized stresses and the deformation of the laminate mid plane can be finally be written as:

Membrane Stresses
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = A \cdot \begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \end{bmatrix} + B \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Bending Stresses
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = B \cdot \begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \end{bmatrix} + D \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

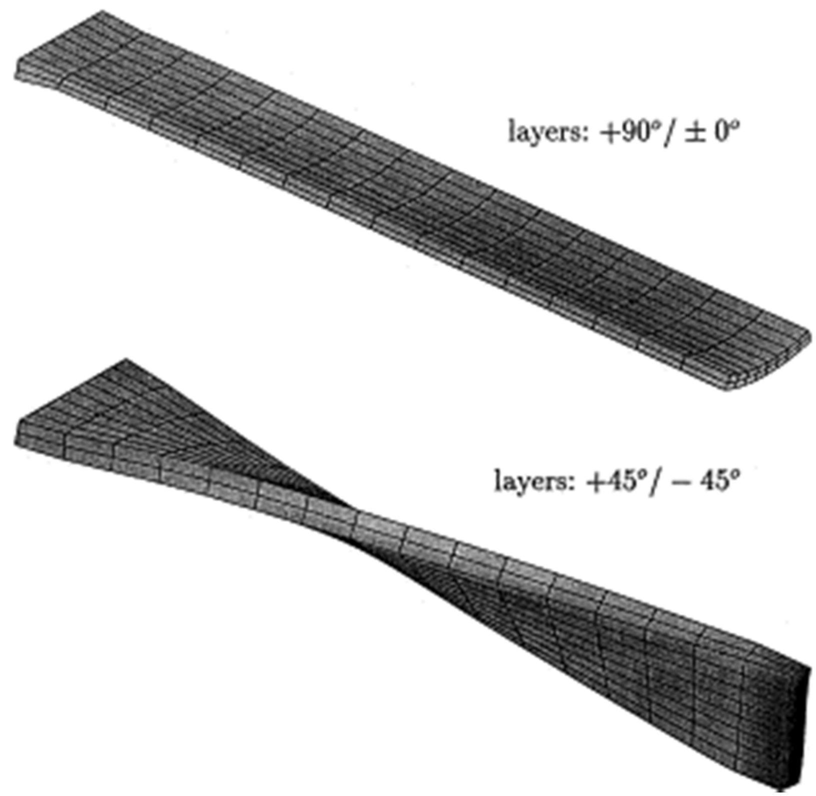
with,

$$A = \int_{-\frac{t}{2}}^{+\frac{t}{2}} C\, dz \qquad B = \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot z\, dz \qquad D = \int_{-\frac{t}{2}}^{+\frac{t}{2}} C \cdot z^2\, dz$$

Generalized Stresses

Which can be written in matrix form as:

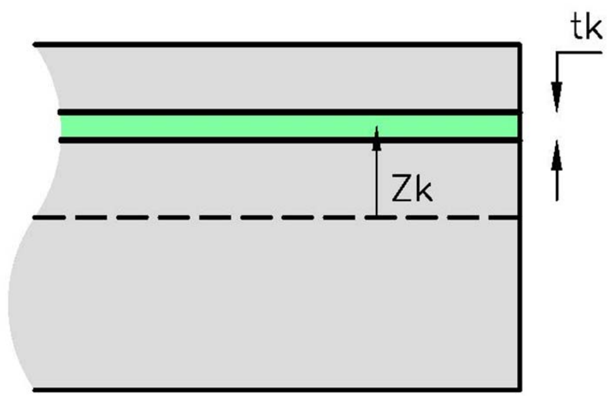
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \cdot \begin{bmatrix} {\varepsilon_x}^0 \\ {\varepsilon_y}^0 \\ {\gamma_{xy}}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$



A is the in plane stiffness matrix. B is the bending stiffness matrix and D is the bending-extension matrix

Generalized Stresses

If this formulation is applied to a laminate made of n different layers, made of material k and with a thickness t_k:



$$A = \sum_{k=1}^{n} C_k \cdot t_k$$

$$B = \sum_{k=1}^{n} C_k \cdot t_k \cdot z_k$$

$$D = \sum_{k=1}^{n} C_k \cdot \left(t_k \cdot {z_k}^2 + \frac{{t_k}^3}{12} \right)$$

Note that if the laminate is symmetric, B = 0. There is no coupling between membrane and bending loads

Membranes

A membrane is a surface element unable of holding flexural forces.

It can be understood as a laminate with a single layer and an extremely small thickness. In this case:

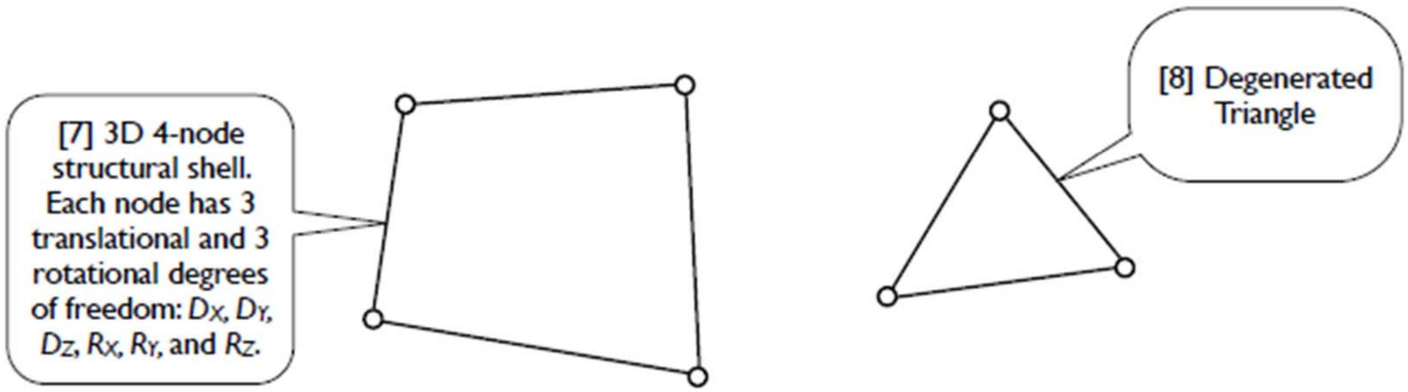
$$A = \sum_{k=1}^{n} C_k \cdot t_k \qquad B = \sum_{k=1}^{n} C_k \cdot t_k \cdot z_k = 0 \qquad C = \sum_{k=1}^{n} C_k \cdot \left(t_k \cdot {z_k}^2 + \frac{{t_k}^3}{12} \right) = 0$$

If a membrane element is well formulated, it is also impossible for it to sustain compression forces: it folds!

Elements in Ansys Workbench

3D Surface Bodies

Element SHELL 181



Although it is possible to define a particular laminate configuration in this element, this option is not provided in Ansys Workbench.

In order to define this element as a laminate, it is necessary to introduce some operation commands to interact with Ansys APDL

Elements in Ansys Workbench

As occurs when dealing with laminates, it is not possible to define the number of layers that have to be integrated along the thickness when SHELL 181 is used in Ansys Workbench.

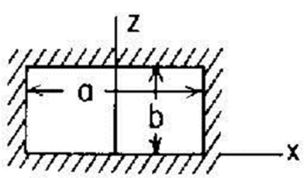
Therefore, it is not possible to define a membrane element unless using external commands.

Numerical Example

To gain a better understanding of the element performance, we will study the case of a rectangular plate clamped along all its edges.

The analytical solution for this case is:

8. Rectangular plate, all edges fixed



8a. Uniform over entire plate

(At center of long edge) $\sigma_{\max} = \dfrac{-\beta_1 q b^2}{t^2}$

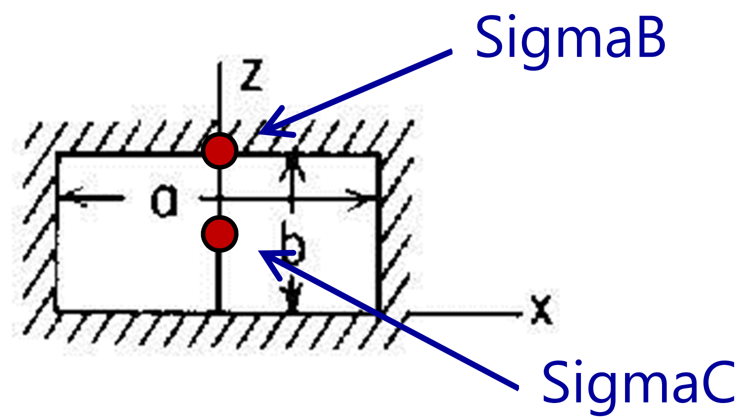
(At center) $\sigma = \dfrac{\beta_2 q b^2}{t^2}$ and $y_{\max} = \dfrac{\alpha q b^4}{E t^3}$

a/b	1.0	1.2	1.4	1.6	1.8	2.0	∞
β_1	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.5000
β_2	0.1386	0.1794	0.2094	0.2286	0.2406	0.2472	0.2500
α	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284

Load applied: q = 5.0 kN/m^2

Numerical Example

The dimensions defined in this example, and the analytical solution obtained is:



a/b	1.8	
b	1.5	m
a	2.7	m
q	5	kN/m2
t	1.5	cm
E	2.00E+05	Mpa

SigmaB	24.36	MPa
SigmaC	12.03	MPa
Ymax	-1.00	mm

Numerical Example

It has been studied the performance of eight different meshes:

Mesh	Nodes
2x3	12
4x7	40
8x14	135
15x27	448
30x54	1,705
60x108	6,649
120x216	26,257
240x432	104,353

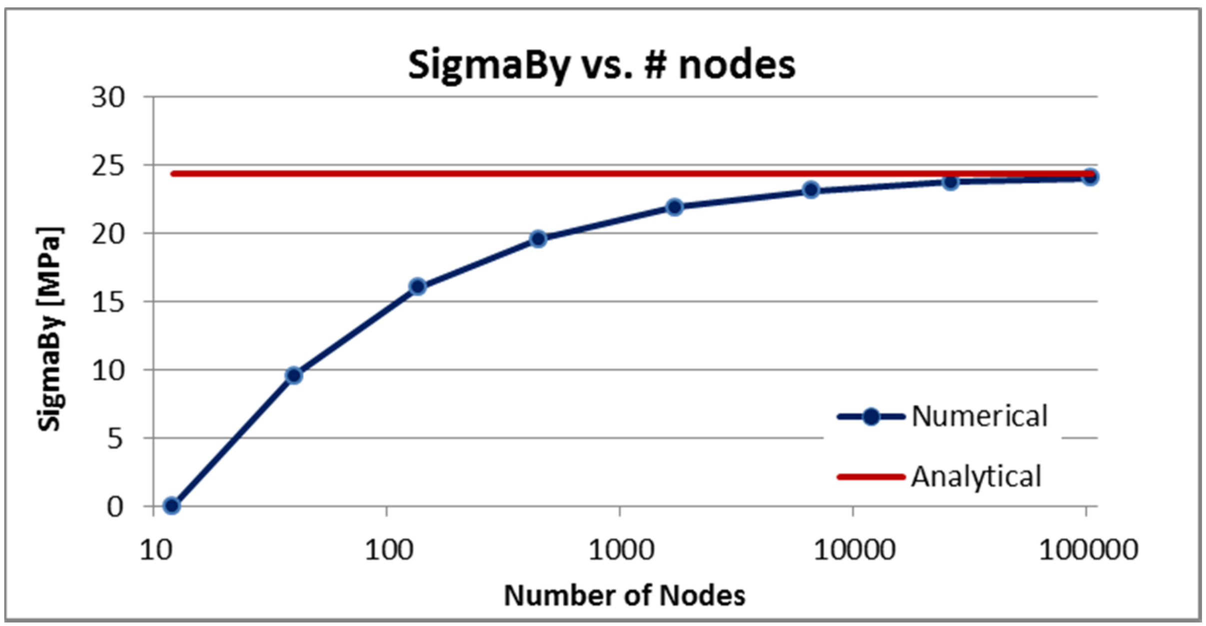
All this meshes are made with quadrilateral elements.

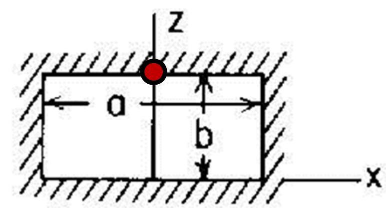
The mesh is structured (mapped) and it follows the grid defined by the coordinate axis.

Numerical Example

The comparison of the numerical results with the analytical ones provide the following graphs:

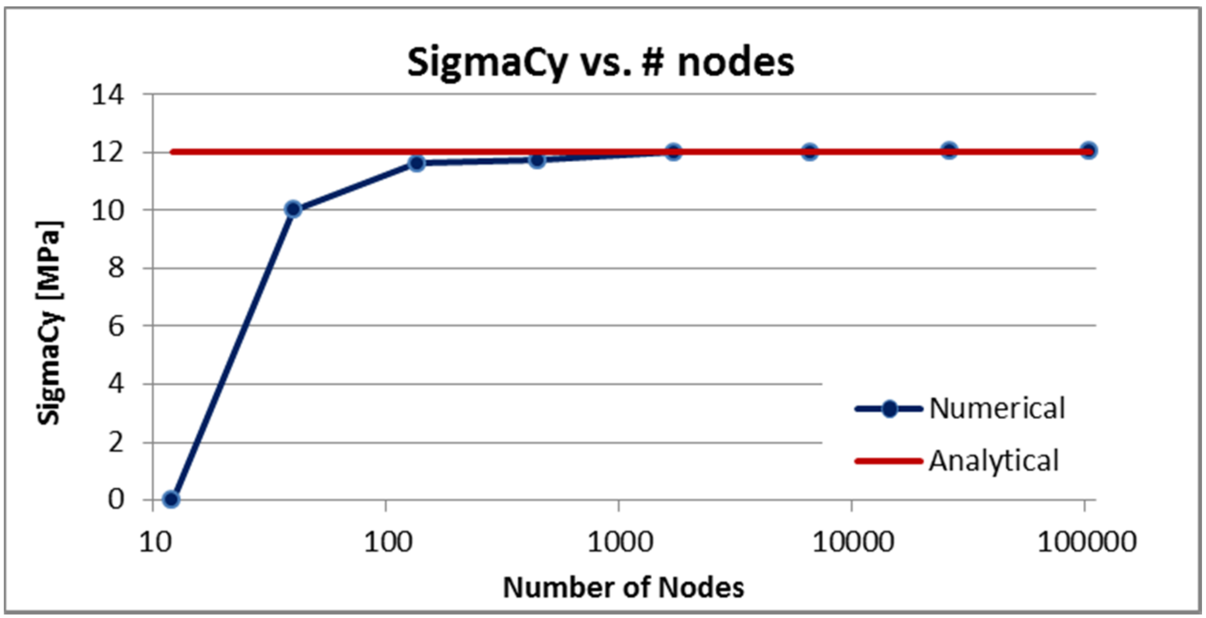
Edge stress vs. # nodes:

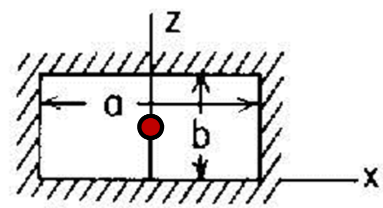




Numerical Example

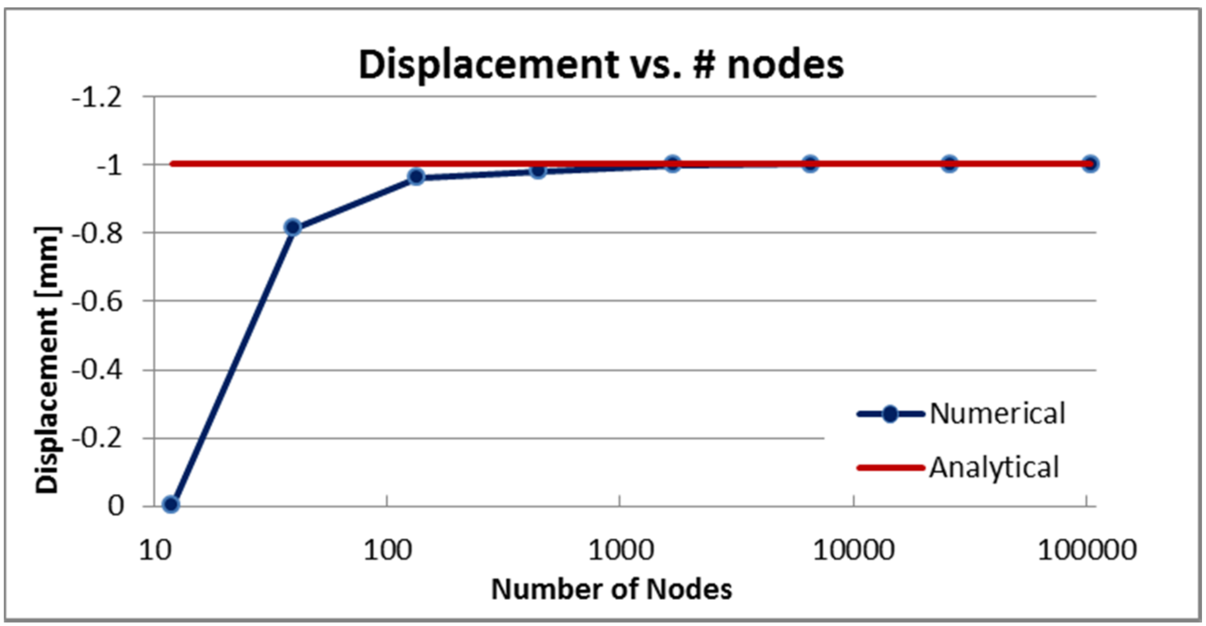
Centre stress vs. # nodes:

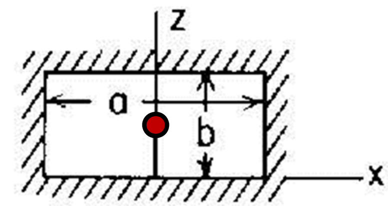




Numerical Example

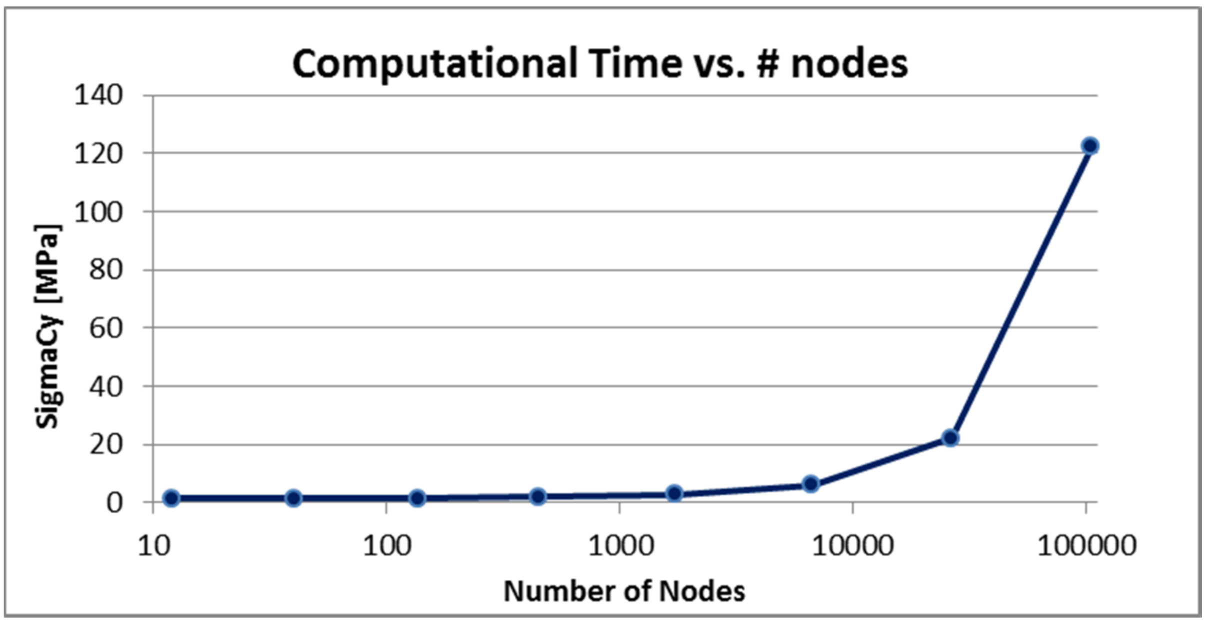
Displacement vs. # nodes:





Numerical Example

Computational Cost vs. # nodes:

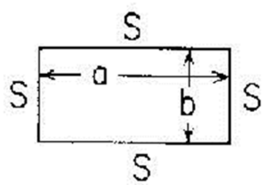


Numerical Example - 2

Solve the case in which this same plate is only simply supported in all its edges.

The analytical solution for this case is:

1. Rectangular plate; all edges simply supported



(At center) $\sigma_{\max} = \sigma_b = \dfrac{\beta q b^2}{t^2}$ and $y_{\max} = \dfrac{-\alpha q b^4}{E t^3}$

(At center of long sides) $R_{\max} = \gamma q b$

a/b	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0	∞
β	0.2874	0.3762	0.4530	0.5172	0.5688	0.6102	0.7134	0.7410	0.7476	0.7500
α	0.0444	0.0616	0.0770	0.0906	0.1017	0.1110	0.1335	0.1400	0.1417	0.1421
γ	0.420	0.455	0.478	0.491	0.499	0.503	0.505	0.502	0.501	0.500