MAE 656 - Advanced Computer Aided Design

04. 2D and 3D Solids – Doc 02

Mesh effects in solid simulations

First remark

The problem that we want to simulate has a unique structural performance; and, therefore, a unique strain-stress field that reproduces accurately the real behavior of the structure.

Any mesh *effect* or mesh *dependency* is an error!

However, as the FEM provides an approximate solution to the problem, the result will have some dependence on the mesh used. We must know these dependencies in order to:

- Avoid them
- Assume them

Mesh effects on the solution

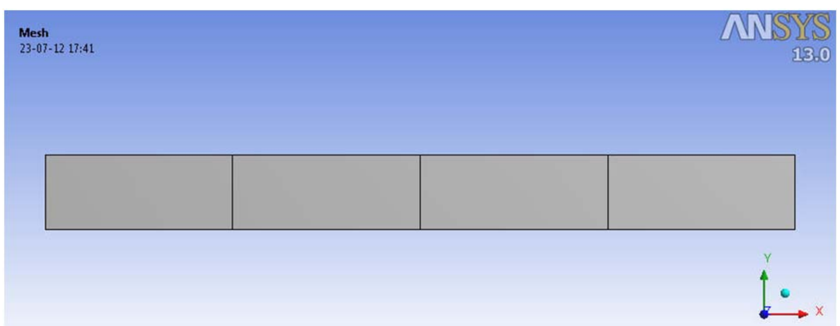
There are several ways in which the mesh may affect the solution of the problem:

1. Reduce result accuracy.

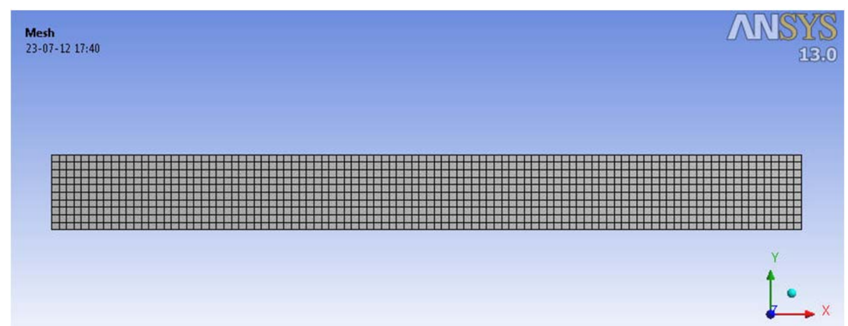
   The solution of the problem is calculated in the nodes defined by the mesh. Less nodes means having less elements where the solution is calculated and, therefore, having a less accurate solution.

   In order to improve the accuracy it is necessary to include as many elements as possible.

Bad mesh



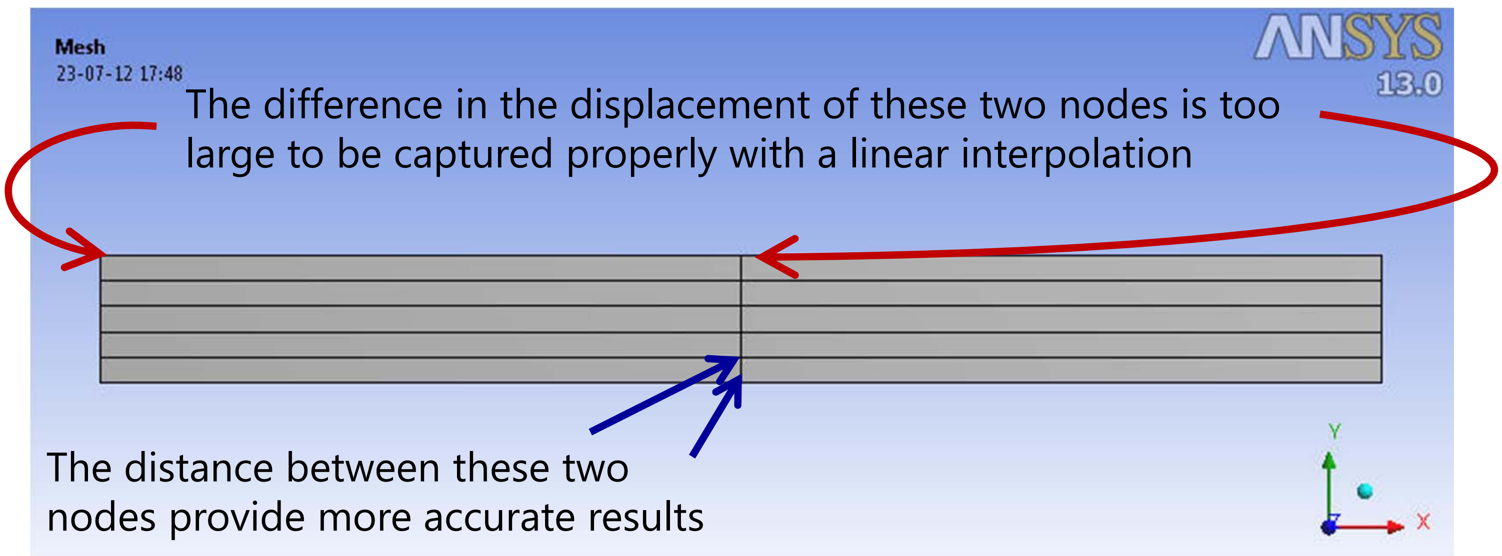
Good mesh



Mesh effects on the solution

In order to improve result accuracy it is also necessary to have the elements as regular as possible.

The FEM calculates the solution in the nodes of the elements. If the element is not regular, we will have regions with solutions that are very closed and regions with the solutions spread apart.



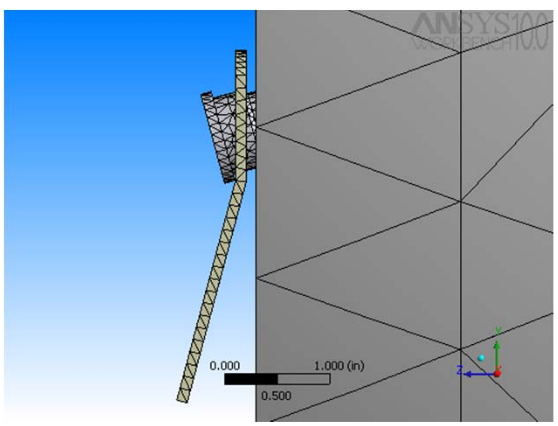
Mesh effects on the solution

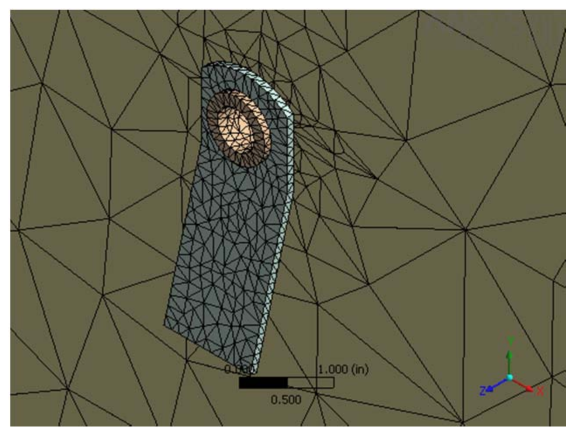
Therefore, we want meshes:

a. As small as possible

b. With the element as regular as possible.

If we presume that there is a region where the field of stress and strains may have larger variations, is a good practice to refine the mesh in that area





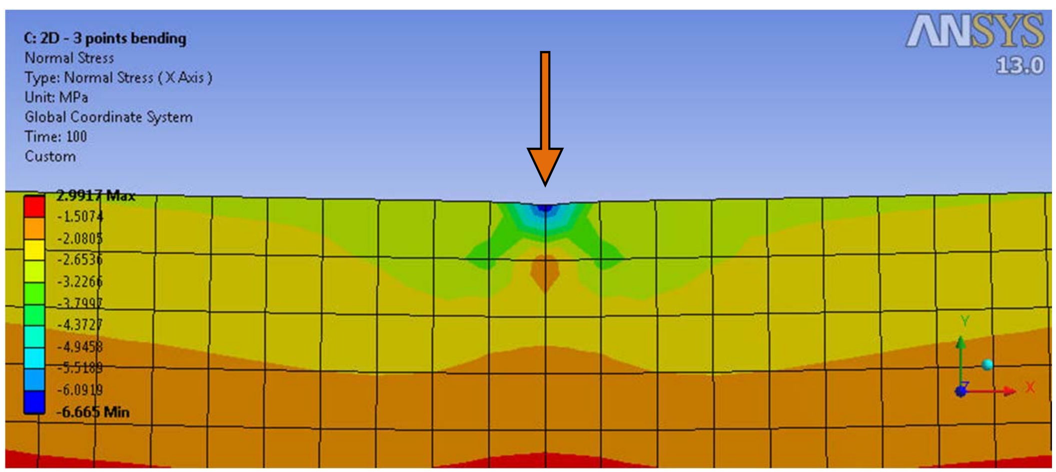
Mesh effects on the solution

2. Stress concentration effects

   Because a numerical simulation can handle any possible load, having small elements can produce large stress concentrations in these elements that may be unrealistic.

   These effects are usually found around points with applied boundary conditions (loads or supports).

   These effects may be also found in sharp edges.



Mesh effects on the solution

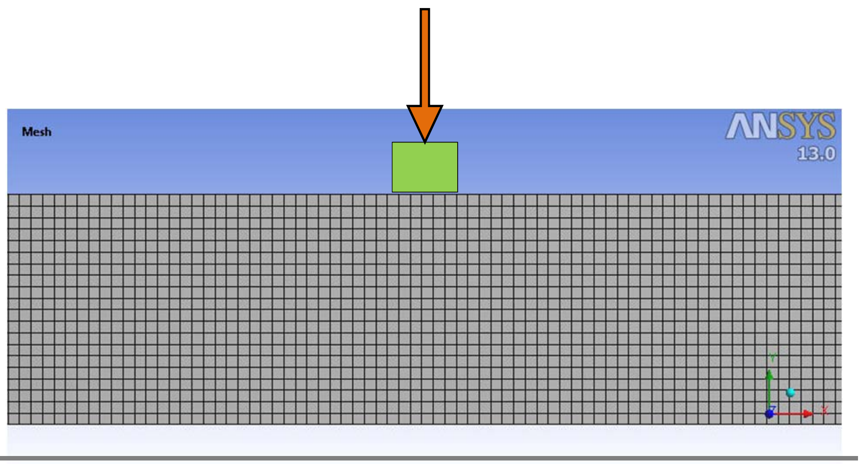
In order to reduce these problems there are some measures that can be adopted:

In boundary conditions (loads and supports):

- It is recommended to load the structure in several points, instead of loading it in a single point. In fact, this is what occurs in real life.
- Another option is to apply the load with another element that can handle these concentration effects

In sharp edges:

- We can smooth them



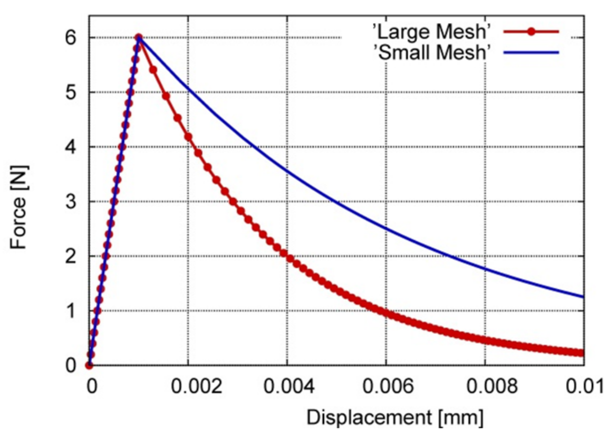
Mesh effects on the solution

3. In non-linear simulations we can obtain incorrect results

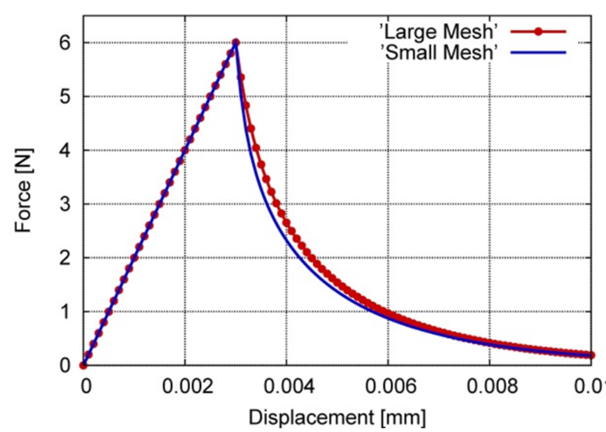
   Material non-linearities have associated energy dissipation effects. The energy dissipated may have some dependency on the mesh.

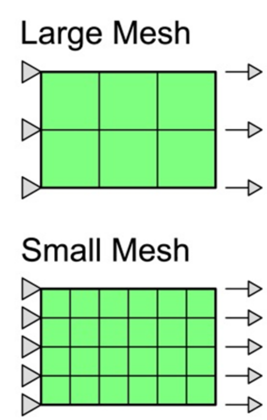
   We have to be sure that the formulation does not have mesh dependency and, if it has it. We have to calibrate the model for a given mesh.

Mesh dependent formulation



Mesh independent formulation





Accuracy vs. Computational cost

As a conclusion we can assess that:

The smaller the mesh, the more accurate will be the results (although we have to be careful with stress concentration effects).

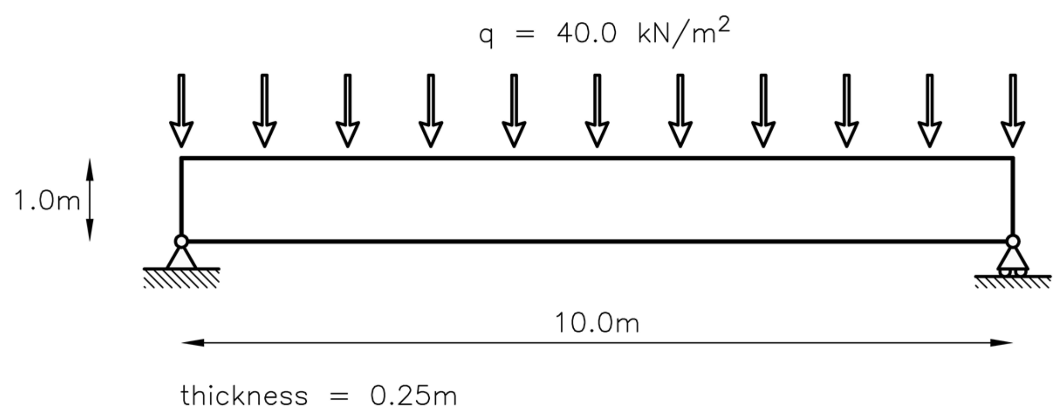
However, smaller meshes imply calculating the solution in more points → increased computational cost!

It is necessary to establish an equilibrium between result accuracy and computational cost.

This equilibrium is defined by the simulation requirements and the computational capacities available.

Solution accuracy - Example

For a better understanding of the implications of having a larger or smaller mesh, let's study the case of a simply supported beam with a distributed load:



The advantage of solving this problem is that we know the exact solution!

Solution accuracy - Example

The most relevant results provided by the analytical solution of the problem proposed are:

$$M_{max} = \frac{q \cdot l^2}{8} = \frac{40 \cdot 0.25 \cdot 10^2}{8} = 125\ kN \cdot m$$

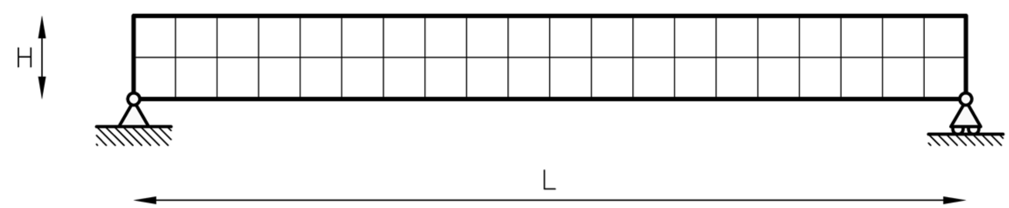
$$\sigma_{max} = -\sigma_{min} = \frac{M}{W} = \frac{M}{b \cdot h^2/6} = \frac{125}{0.25 \cdot 1.0^2/6} = 3000\ kN/m^2 = 3.0\ MPa$$

$$\delta_{max} = \frac{5 \cdot q \cdot l^4}{384 \cdot E \cdot I} = \frac{5 \cdot 40 \cdot 0.25 \cdot 10^4}{384 \cdot 2.0 \cdot 10^8 \cdot 0.25 \cdot 1.0^3/12} = 3.125 \cdot 10^4\ m = 0.3125mm$$

Solution accuracy - Example

These results will be compared with the results obtained with several FEM simulations in which we vary the mesh used.

Defining the mesh by the number of elements in along the thickness (H) and length (L)



In this case, the mesh is 2x20

Solution accuracy - Example

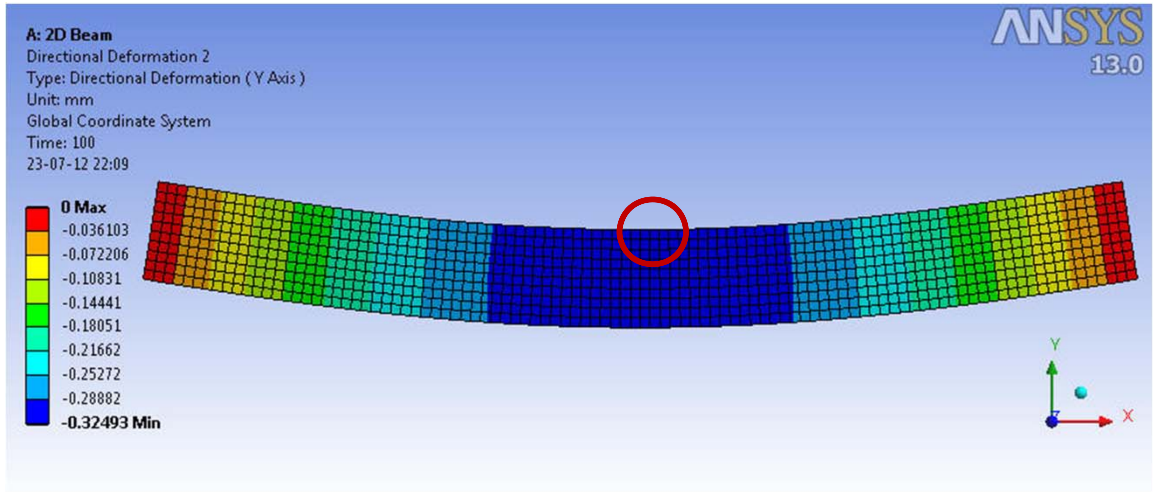
We have defined seven different meshes:

Mesh name	# of elements	# of nodes
Mesh 1x4	4	10
Mesh 1x10	10	22
Mesh 2x20	40	63
Mesh 3x30	90	124
Mesh 4x40	160	205
Mesh 10x100	1000	1111
Mesh 50x500	25000	25551

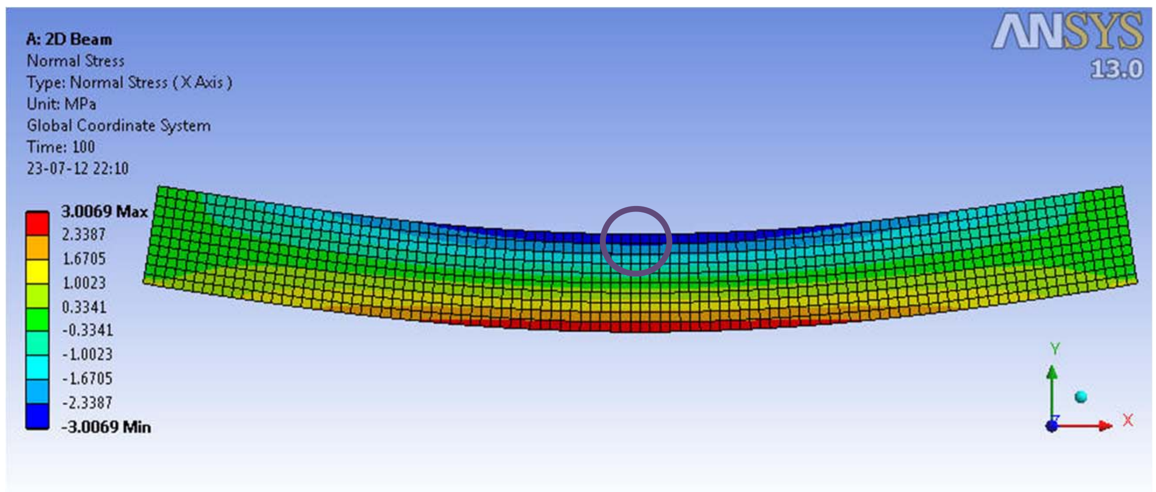
Solution accuracy - Example

Results studied:

Displacement in ◯ dot

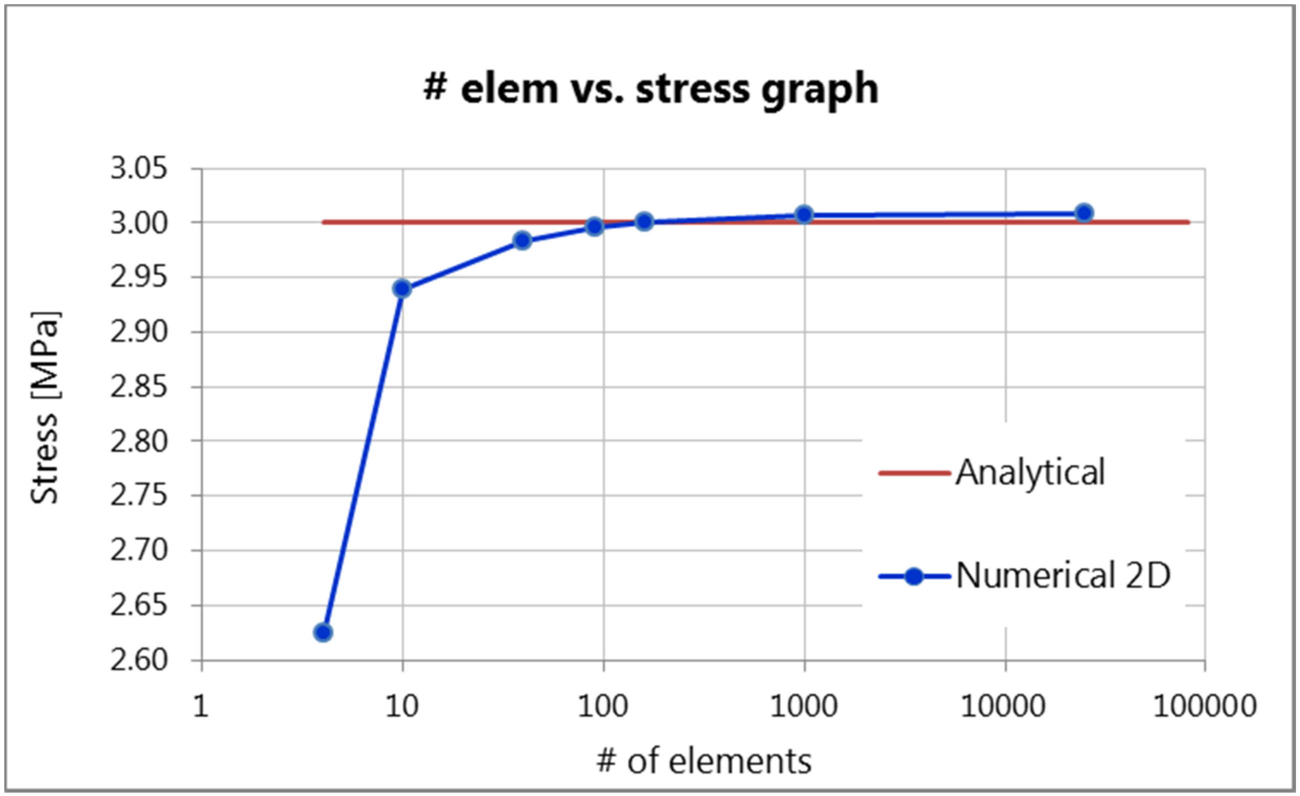


Stress in ◯ dot



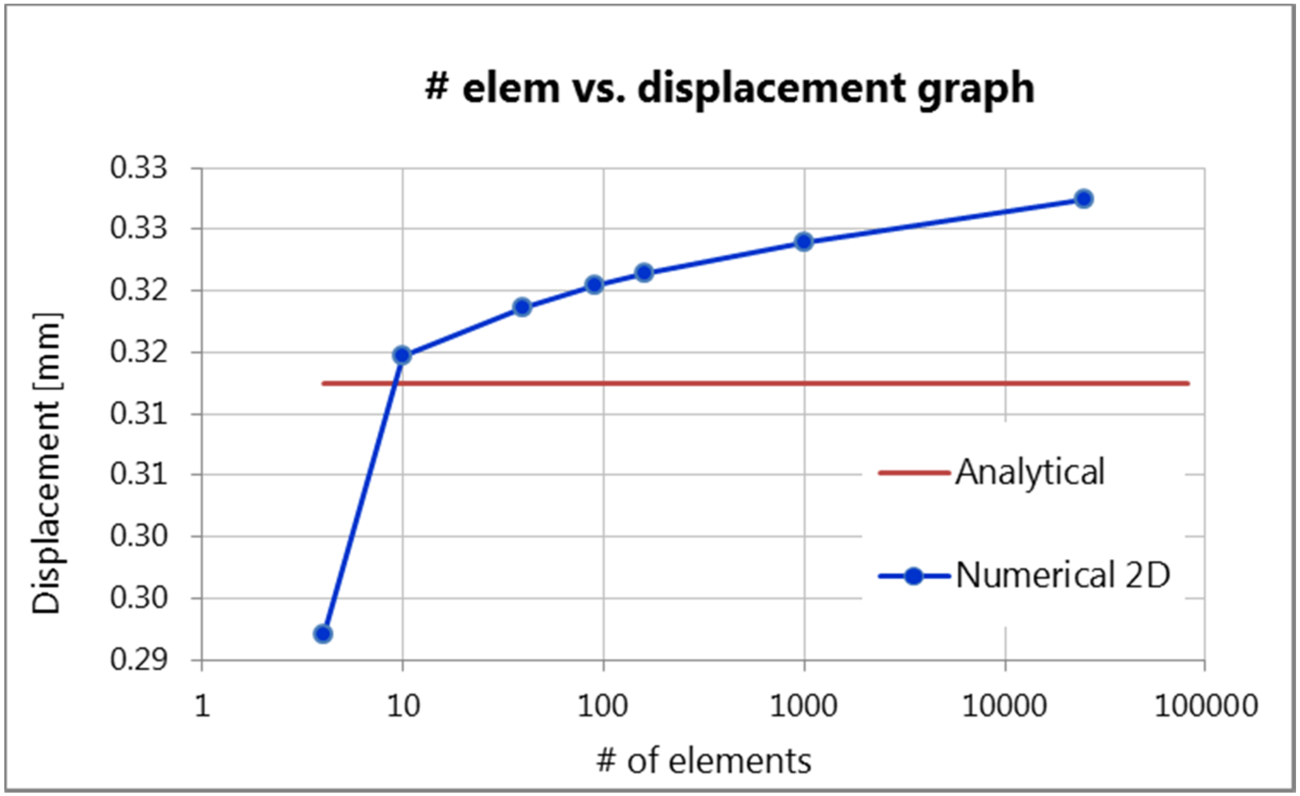
Solution accuracy - Example

Stress accuracy:



Solution accuracy - Example

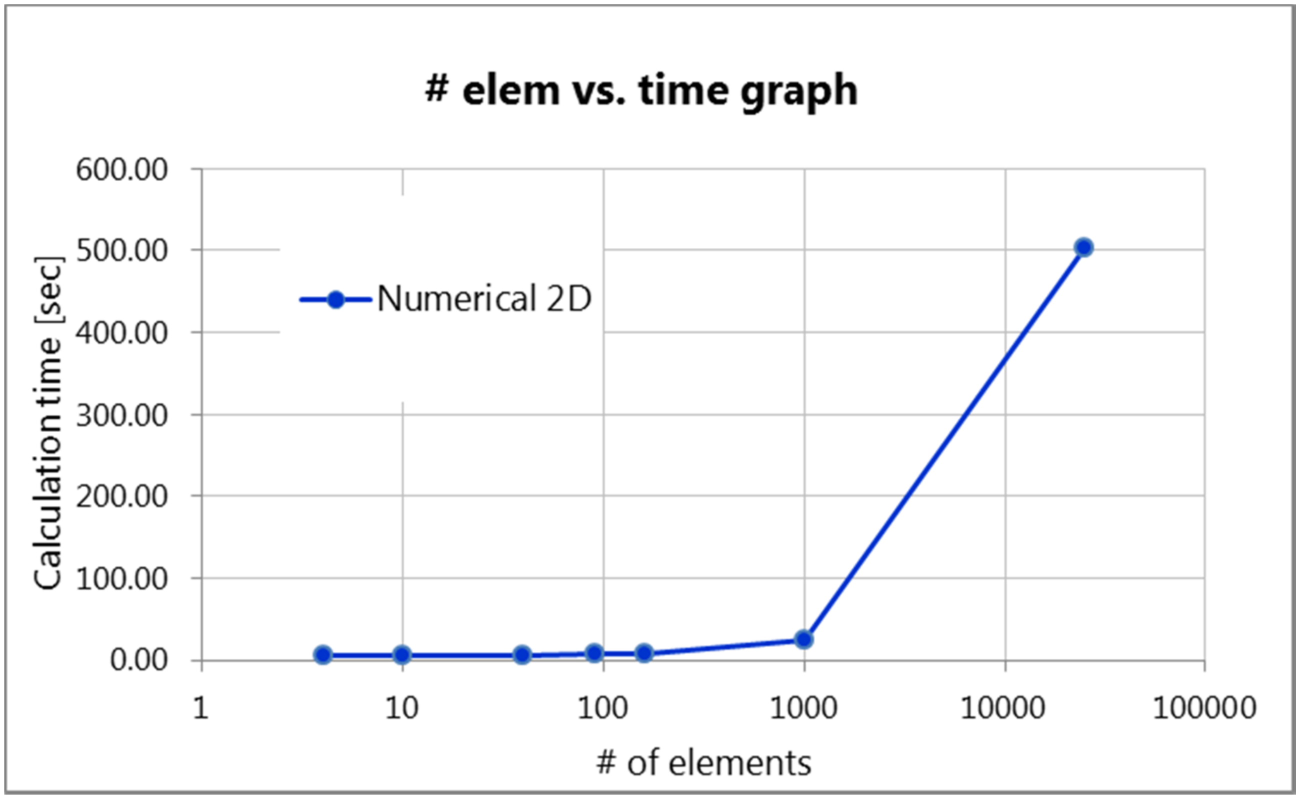
Displacement accuracy:



The reason for the difference found between the analytical and the numerical result is that the beam is quite high and it does not verify the Euler-Bernoulli hypothesis (a better correlation is obtained if the cross section of the beam is defined as 0.25x0.25)

Solution accuracy - Example

Computational cost:



Solution accuracy - Example

Does a 3D simulation improve the result accuracy?

Mesh name	# of elements	# of nodes
Mesh 1x1x4	4	20
Mesh 1x1x10	10	44
Mesh 1x2x20	40	126
Mesh 1x3x30	90	248
Mesh 1x4x40	160	410
Mesh 2x10x100	2000	3333
Mesh 8x32x320	81920	95337

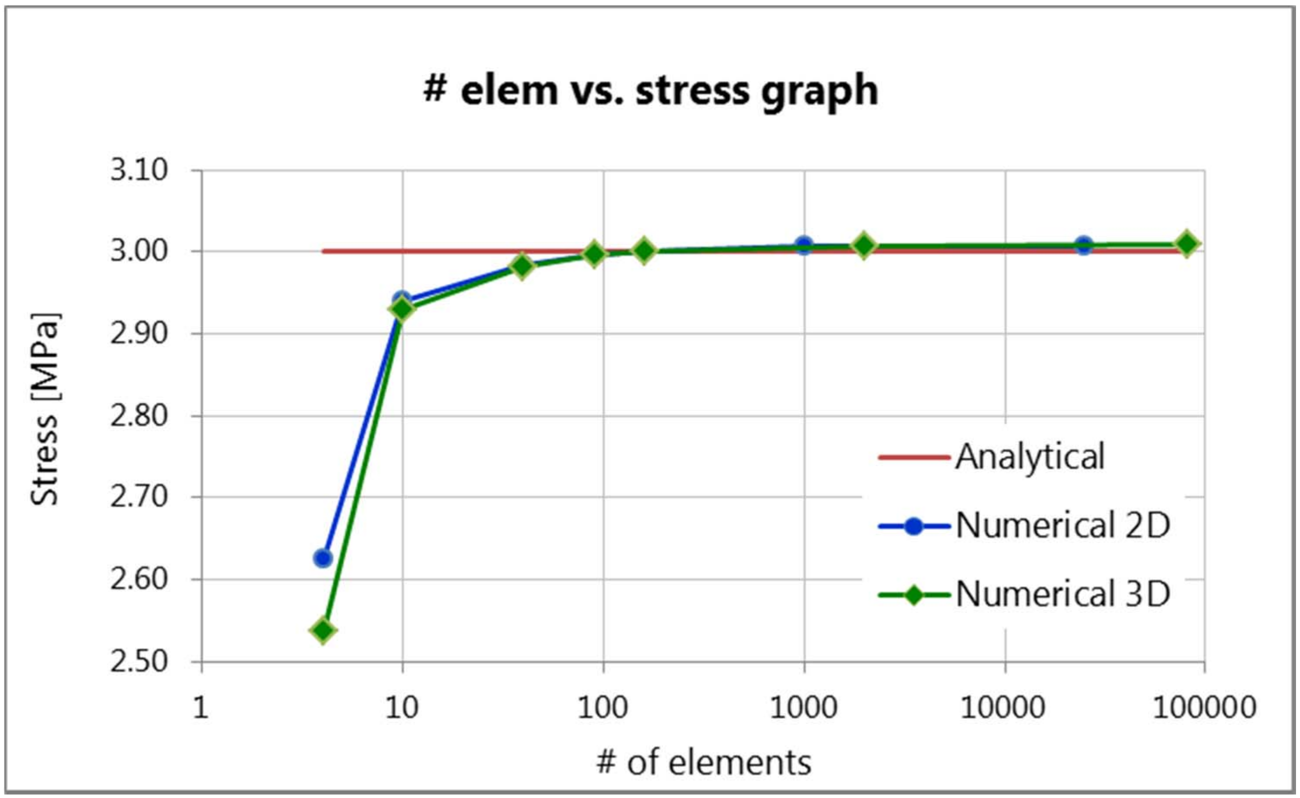
If we compare the degrees of freedom of 2D vs 3D in the smaller mesh:

2D = 2x25,551 = 51,102dof

3D = 3x95,337 = 286,011 dof
5.6 times 2D !!!

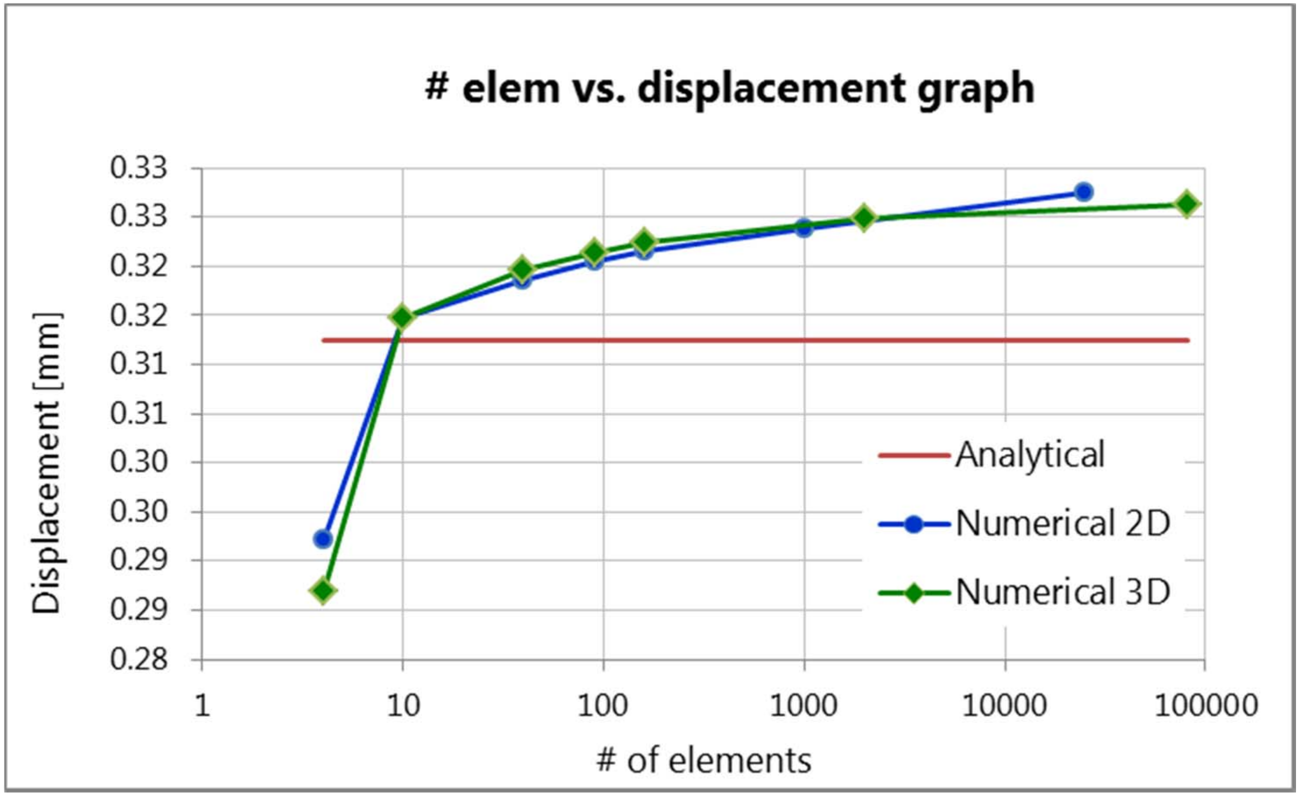
Solution accuracy - Example

Stress accuracy:



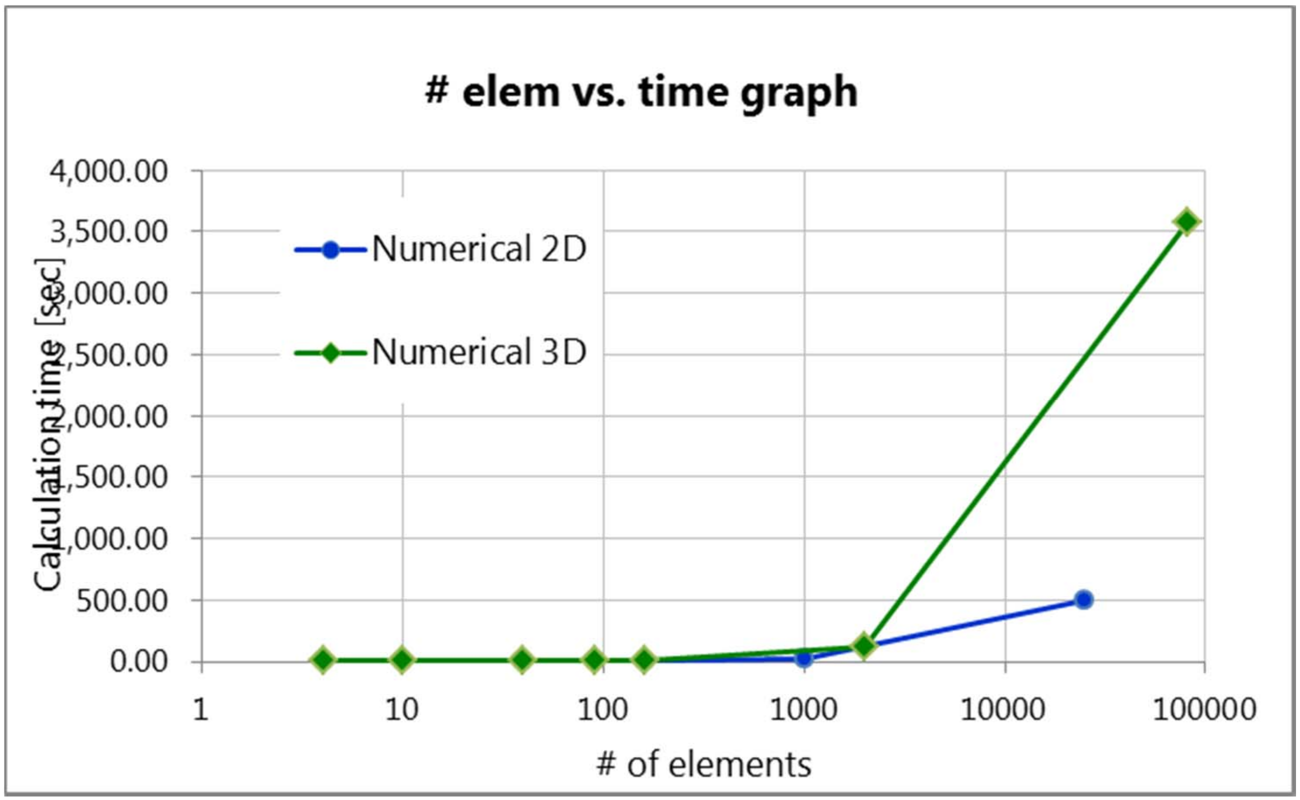
Solution accuracy - Example

Displacement accuracy:



Solution accuracy - Example

Computational cost:



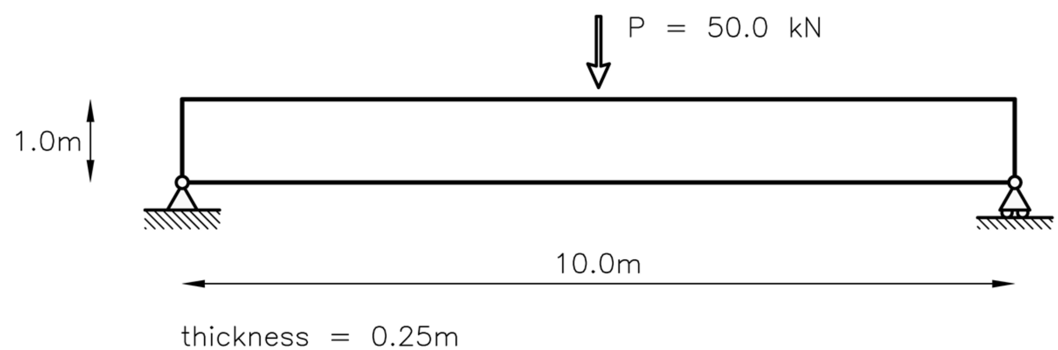
Example Conclusion

In current simulation:

- A mesh with 90 (3x30) or 160 (4x40) elements is more than enough to obtain an accurate result. Smaller meshes do not provide better results and the computational cost is increased enormously
- A 3D simulation does not make sense in this case. It does not improve accuracy and it increases the computational cost.

Stress concentration - Example

To study the effect of stress concentration we will study the following example



We have to obtain the following graphs for this structure:

upper stress
lower stress
displacement
computational time

Solution accuracy - Example

The analytical results in this case are:

$$M_{max} = \frac{P \cdot l}{4} = \frac{50 \cdot 10}{4} = 125\ kN \cdot m$$

$$\sigma_{max} = -\sigma_{min} = \frac{M}{W} = \frac{M}{b \cdot h^2/6} = \frac{125}{0.25 \cdot 1.0^2/6} = 3000\ kN/m^2 = 3.0\ MPa$$

$$\delta_{max} = \frac{P \cdot l^3}{48 \cdot E \cdot I} = \frac{50 \cdot 10^3}{48 \cdot 2.0 \cdot 10^8 \cdot 0.25 \cdot 1.0^3/12} = 2.5 \cdot 10^4\ m = 0.25mm$$