1. ABSTRACT

A numerical model implemented is proposed for the simulation of the urinary bladder. For the implementation, a stress-strain relationship is specified based on the evaluation of the strain energy function. The constitutive model is based on a hyperelastic neo-Hookean model. The analysis is treated within a 3D framework in total Lagrangian description and a quasi-incompressible element. The interaction of the bladder wall with urine is modelled via the Particle Finite Element Method (PFEM) [1,2]. The PFEM allows the reproduction of filling and voiding of the bladder with urine, accounting for the wall-fluid interaction effects.

2. INTRODUCTION

The objective is to simulate the main mechanical part of the bladder: the detrusor, a smooth muscle which is responsible for maintaining an almost constant pressure inside the bladder during the filling and the storage of the urine [3].

Smooth muscles are known by its nonlinear behaviour, and in the specific case of the detrusor the change in the stiffness is promoted not only by its mechanical properties but also by chemical reactions. The detrusor is innervated by an autonomic nervous system that allows the muscle to be partly contracted, maintaining tonus for prolonged periods with low energy consumption.

Given the complexity of biological materials and its multi-scale hierarchy, some simplifications were made and the classical nonlinear continuum mechanics theory was applied. The proposed model is based on the three dimensional representation of the detrusor tissue by basically two different structures: a hyperelastic matrix, representing the extracellular substance, and viscoelastic fibres, representing the passive fibres [4]. For the sake of simplicity the chemical reactions were not taken into account in this model. The homogenization theory is considered to represent the integration of these two main structures.

To represent the mechanics of the bladder, the constitutive equations here presented are written in a total Lagrangian approach [5], were the unknown displacements are described in the reference configuration.
In short, the model considers the implementation of a Hyperelastic-Viscoelastic constitutive law in a finite element code allowing for Fluid Structure Interaction (FSI) [6] effect for the simulation of bladder wall and urine. The problem is solved by Newton-Raphson method with Bossak Scheme.

State of art:

The simulation of the urinary bladder still is a challenge to be overcome. The first intend models considered hyperelastic constitutive models. The scientific community realized the necessity to implement more complex models to simulate this organ and accurately represent its ability to store urine under low pressure due to relaxation of its musculature. Viscoelastic models were introduced in the following years. More refined models with combined elasticity and viscoelasticity were considered. A homogenized model to represent the multi-scale hierarchy of elements in the smooth muscle tissue is proposed. The combination of hyperelasticity and viscoelasticity has been put in place in recent years [7-10].

3. CONSTITUTIVE MODEL

The proposed model is based on the representation of the detrusor tissue by basically two different structures: a hyperelastic matrix, representing the extracellular substance, and viscoelastic fibres, representing the passive fibres. For the sake of simplicity the chemical reactions are not taken into account in this model.

A stress-strain relationship is specified based on the evaluation of the strain energy function, in terms of classical non-linear continuum mechanics.

The hyperelastic model, based in a neo-Hookean formulation, was put in place and followed by validation using benchmark experiments. Incompressibility is treated using an Ogden formulation for quasi-incompressible materials [11].

The combined formulation of hyperelasticity and viscoelasticity is being implemented through the volume fraction method [4].

4.1 - The Hyperelastic constitutive model

The hyperplasic quasi-incompressible constitutive model was implemented in the object oriented code Kratos [12] to describe the behaviour of biological and rubber-like materials.

To represent hyperelasticity, the Neo-Hookean model was chosen, with the following equation for the isochoric part of the strain energy potential and the stress respectively:

\[ W^{\text{iso}} = \frac{1}{2} \mu (J^3 C_{kk} - 3) \]

\[ S_{\text{iso}} = \frac{\partial W^{\text{iso}}}{\partial E_y} = \mu J^3 (\delta_{yy} - \frac{1}{3} C_{kk} C_{yy}) \]

The tangent matrix, in the hyperelastic case is the lagrangian elasticity tensor that comes from

\[ DS[u] = C : DE[u] \]

\[ C = \frac{\partial S}{\partial E} = 2 \frac{\partial S}{\partial C} = 4 \frac{\partial^2 W}{\partial C \partial C} \]
The final form of the isochoric part of the elasticity tensor is given by:

$$
C_{iso} = \frac{1}{3} \mu J^{-3} I C_{ij}^{-1} + \left( \frac{1}{9} \mu J^{-3} \right) C_{ij}^{-1} C_{ik}^{-1} \left( \frac{1}{2} C_{jk}^{-1} C_{jl}^{-1} + C_{jl}^{-1} C_{jk}^{-1} \right) \quad (I)
$$

To treat incompressibility, Ogden model [11] for quasi-incompressible, rubber-like materials was implemented. In this model we consider a shear modulus high enough to approach the quasi-incompressibility condition.

The volumetric part of the strain energy is given by:

$$
W_{vol} = k G(J) \quad G = \beta^{-2} \left( \beta \ln J + J^{-\beta} - 1 \right) \quad , \quad \beta = 9
$$

The volumetric part of the stress is computed as

$$
S_{vol} = 2 \frac{\partial W_{vol}(J)}{\partial C} = J p C^i
$$

Where the hydrostatic pressure $p$ is described below, we get the volumetric part of the stress as

$$
p = \frac{dW_{vol}(J)}{dC} = k \frac{dG(J)}{dJ} \quad S_{vol} = k \frac{1}{\beta} (1 - \frac{1}{j^\beta}) C^i
$$

And the volumetric part of the elasticity tensor as:

$$
C_{vol} = \frac{\partial S_{vol}}{\partial E} = 2 \frac{\partial S_{vol}}{\partial C} = 4 \frac{\partial^2 W_{vol}}{\partial C^2 C}
$$

Finally we get:

$$
C_{ijkl} = \frac{k}{2} \left( \frac{11}{2} J^{-\frac{5}{2}} - J^{-9} \right) C_{ijkl}^{-1} + \frac{k}{2} (1 - J^{-9}) \left( \frac{1}{2} C_{ik}^{-1} C_{jl}^{-1} + C_{jl}^{-1} C_{ik}^{-1} \right) \quad (II)
$$

The final form of the elasticity tensor is given by adding up (I) and (II):

$$
C = C_{iso} + C_{vol}
$$

$$
C_{ijkl} = -\frac{1}{3} \mu J^{-3} I C_{ijkl}^{-1} + \left( \frac{1}{9} \mu J^{-3} \right) C_{ijkl}^{-1} C_{ik}^{-1} \left( \frac{1}{2} C_{jk}^{-1} C_{jl}^{-1} + C_{jl}^{-1} C_{jk}^{-1} \right) + \left( \frac{11}{2} J^{-\frac{5}{2}} - J^{-9} \right) C_{ijkl}^{-1} C_{ik}^{-1} + \frac{k}{2} (1 - J^{-9}) \left( \frac{1}{2} C_{ik}^{-1} C_{jl}^{-1} + C_{jl}^{-1} C_{ik}^{-1} \right)
$$

5. PRELIMINARY TESTS

In this section we present some preliminary tests for the hyperelastic constitutive model implemented in Kratos [12]. These tests intend to validate the accuracy of the model and the capability of the formulation to account for the FSI [6] during the bladder filling process.
5.1 – Inflation of a quasi-incompressible rubber sphere

The inflation of a quasi-incompressible rubber sphere under internal pressure is presented. The sphere inflation was simulated using the hyperelastic neo-Hookean model implemented. For the sake of simplicity, considering symmetry, only half-sphere was considered [13-14].

The inflation pressure is a function of Cauchy-stress, the radius and thickness of the sphere, as shown below:

\[ p_1 = 2 \frac{h}{r} \sigma \]

From the constitutive equations, taking into account incompressible material, we obtain the single relation between the stretch in the principal direction (or circumferential stretch) and the associated circumferential Cauchy stress:

\[ \sigma = \sum_{p=1}^{N} \mu_p (\lambda^{\alpha_p} - \lambda^{-2\alpha_p}) \]

Where \( N = 1 \), \( \alpha_p = 2 \) and \( \mu = \mu_1 = 624.0 \) KPa for the neo-Hookean model. The geometrical data considered are: initial internal radius: 0.03 m and initial thickness: 0.005 m. The finite element mesh consists of 3,278 tetrahedral elements and 675 nodes. Fig 1. represents the sphere inflation: with the results of Z displacement in respect to the reference mesh.

![Fig 1 – Inflated half-sphere is plotted in comparison with the original geometry (in green)](image)

In Fig.2 is plotted Cauchy-Stress versus stretch \( \lambda \), considering bulk modulus of 10,000 KPa and the analytical response for incompressible rubber.

![Fig 2 - Cauchy-Stress versus stretch \( \lambda \)](image)
The inflation of half-sphere under internal pressure provides a first approximation of bladder inflation.

5.2 - Half-sphere fluid interaction

In this section we simulate the filling of a half-sphere with fluid considering fluid-structure interaction, see Fig 3.

The material considered for the half-sphere is the neo-Hookean quasi-incompressible rubber, shear modulus of 100.0 KPa and a bulk modulus of 1,000.00 KPa and geometry 0.03 m internal radius and 0.005 m thickness.

The fluid considered is an incompressible fluid, written in an updated Lagrangian formulation [18].

6. WORK IN PROGRESS

Our goal is to simulate the human bladder under filling and voiding conditions. To achieve this goal, we start with the geometry from a simplified high-resolution three-dimensional model of the human bladder of a 39 year old man, with no known urological diseases. The morphology came from the Visible Human Project (VH), and the input geometry from the work of Zbyněk Tonar et al [15]. This geometry has been treated with the pre-post processor GID [16]. The initial mesh size is of the order of 1 million tetrahedral elements (see Fig 4).
The structural element used in the simulation is written in a total lagrangian description and it’s represented as quasi-incompressible 4 nodes tetrahedral [16]. This formulation is used in combination with the Particle Finite Element Method (PFEM) [18] for solving the bladder and urine Fluid Structure Interaction (FSI) simulation. The viscoelastic Kelvin model is under implementation.

8. REFERENCES:

16. GID Pre-Post Processor: www.gidhome.com